Second Final exam: **Electrodynamics of Radiation Processes,**
12th April 2019, 9-12
The exam consists of 80 points in total.

Write your name and student ID number on every page.
Make certain to clearly label which answer is which on your exam papers.

Only calculators can be used. **No laptops, tablets, iPads, or other internet devices are allowed. Also no books or notes are allowed.**

Explain clearly all of the steps that you use to derive all your results. If you are not sure of a particular step make an estimate, be clear you doing this, and continue.

Make certain that your handwriting is readable to someone besides yourself.

1. **Blackbody Radiation** [**20 pts**]

   (a) [**6 pts**] Explain what is meant by blackbody radiation, and given an astrophysical example of (almost perfect) blackbody radiation. Explain why it is difficult to find a perfect blackbody.

   (b) [**4 pts**] Given the Planck law:

   $$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\exp(\frac{h\nu}{kT}) - 1}$$

   derive the Rayleigh-Jeans law ($h\nu \ll kT$) and the Wien law ($h\nu \gg kT$).

   (c) [**6 pts**] Show that the Wien displacement law, derived from the Planck law, is:

   $$\nu_{max} = 5.88 \times 10^{10}\mathrm{T} \ \ \mathrm{Hz}$$

   Make clear the steps in your derivation.
   [Hint: $x = 3(1 - e^{-x})$ has the approximate root, x=2.82]

   (d) [**4 pts**] The two brightest stars in the Orion constellation are Betelgeuse with a surface temperature T $\sim$ 3400 K and Rigel with T $\sim$10 100 K. Estimate the frequency at which the peak of the emission occurs for both these stars, and make a sketch of the spectrum of both stars on the same flux vs. frequency plot. Make clear the position of both spectra relative to each other.

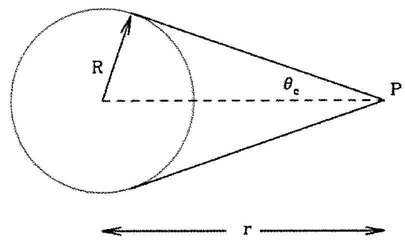

Figure 1: An observer at P sees a spherical source as a disc of angular radius θ_c, where the intrinsic radius of the source is R, and it is at distance r from an observer at P.

2. **Radiative Transfer [20 pts]**

   (a) **[4 pts]** How is a solid angle defined in spherical coordinates? Why is this concept important in describing radiation processes?

   (b) **[6 pts]** Assuming we only see the surface of a dense spherical cloud, with a constant intensity (see Figure 1), calculate the flux in terms of intensity if the distance of the cloud, r, from the observer is much greater than the intrinsic radius of the cloud, R ($r \gg R$). How is the result different for r $\approx$ R?

   (c) **[3 pts]** Explain what is meant by optical depth.

   (d) **[5 pts]** Explain the radiative transfer equation, both what the terms in the equation mean and how and when the equation itself is used:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

   (e) **[2 pts]** Give the solution to this equation, resulting in a general expression for specific intensity, in the case of emission only.

3. **Compton Scattering [10 pts]**

   (a) **[5 pts]** Explain the basic principles of Compton scattering and inverse Compton scattering, and the properties of the resulting radiation.

   (b) **[5 pts]** Explain what is the Sunyaev-Zeldovich effect. This should include what astrophysical effect it alters under which conditions, and what can be learnt from a careful study of this effect.

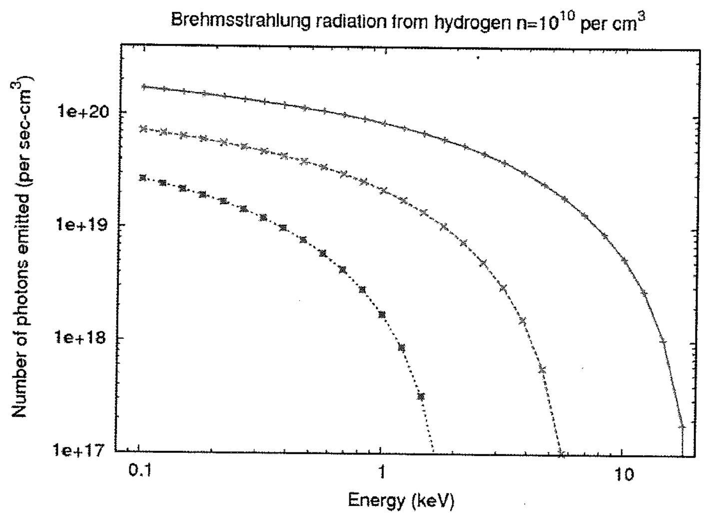

Figure 2: X-ray spectra of three different Bremsstrahlung sources.

4. **Bremsstrahlung [20 pts]**

   (a) **[4 pts]** Explain the circumstances in which Bremsstrahlung (also called free-free) radiation occurs. Give an example of an astrophysical source of Bremsstrahlung radiation, explaining how the physical conditions arise.

   (b) **[4 pts]** Explain what is the impact parameter, b, for Bremsstrahlung radiation. Use a sketch in your explanation. What determines the lower limit to this parameter?

   (c) **[2 pts]** How are the properties of the impact parameter, b, included in the Bremsstrahlung emissivity equation?

   (d) **[5 pts]** Assume that Bremsstrahlung sources are optically thin uniform spheres of fully ionised hydrogen, with gas mass M_g, temperature T, and radius R. Determine the expression for the total Bremsstrahlung luminosity of these sources in terms of these parameters, remembering that Bremsstrahlung emissivity is defined by: $\epsilon^{brems} = 1.4 \times 10^{-27} Z^2 n_e n_i T^{1/2} \bar{g}_B$
   [Hint: Express the proton and electron densities in terms of M_g.]

   (e) **[5 pts]** In Figure 2 the X-ray spectra of three different Bremsstrahlung sources are shown, what is the temperature range covered by these three cases? which source is likely to be the most massive of the three?
   [Note: The Boltzmann constant, $k = 8.6 \times 10^{-5}$ eV K^{-1}.]

5. **Synchrotron Radiation** [10 pts]

(a) **[2 pts]** Explain which astrophysical objects produce Synchrotron radiation, and which physical processes make the necessary conditions possible.

(b) **[3 pts]** What is the typical form of the observed spectrum of Synchrotron radiation? Make a sketch of the flux emitted as a function of frequency, ν, and give as much detail as you can on the form of the flux distribution.

(c) **[2 pts]** What determines the slope of the spectral energy distribution of a synchrotron spectrum?

(d) **[3 pts]** Explain the effect of the cooling time of an electron on the shape of synchrotron emission spectrum at high frequencies. How is the cooling time determined?